OSCILLATIONS IN AN INHOMOGENEOUS MEDIUM

AND THE NATURE OF SPINNING GAS-FILM DETONATION

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1. Acoustic resonance oscillations in the products of the detonation of homogeneous gaseous mixtures were first analyzed in [1-4], where it was pointed out that the experimentally measured spin frequency of the "head" ω_{es} agreed closely with the frequency of the first harmonic ω_{11} of the transverse oscillations of a cylindrical column of gas with constant parameters, corresponding to a state in the Jouguet plane. The perturbations of the flow were studied in [1-4] without taking into account the existence of a reaction zone. Since the disturbances propagate from the reaction zone downstream and do not penetrate from the supersonic region through the Chapman–Jouguet surface to the front, in these works the questions of why the detonation front is itself unstable and why $\omega_{es} \approx \omega_{11}$ are now answered.

A qualitative study of the stability of a plane detonation wave with random curvature of the combustion front was carried out in [5, 6] for a model of a detonation with a period of induction followed by an instantaneous reaction.

The problem of the stability of a detonation front with an adjacent one-dimensional reaction zone was solved in [7-9] in a more accurate mathematical formulation for gaseous mixtures. It was pointed out that an overdriven detonation wave remains stable as the liberation of heat behind its front approaches zero [7, 9]. The stability of the main solution of the equations of hydrodynamics and chemical kinetics for a Chapman–Jouguet detonation relative to small perturbations was studied in [8], where some of the characteristic numbers were obtained and it was found that the frequencies of the cylindrical harmonics which grow with the passage of time depend on the ratio of the width of the reaction zone to the radius of the tube.

An acoustic model, presenting a criterion for the development of spontaneously arising transverse waves, was proposed in [10], and it was shown that for any reasonable kinetics infinitesimal high-frequency transverse disturbances are amplified in the exothermal reaction zone of a Chapman–Jouguet detonation. The theory in [11] describes the mechanism of the earliest behavior of the wavefront during the buildup of the transverse waves and shows that finite-amplitude waves will not appear in flames or shock waves accompanied by an endothermal reaction zone.

Stationary spinning detonation states in a heterogeneous system consisting of a gaseous oxidizer in the volume of a pipe and a film of liquid fuel on the pipe walls were observed and studied experimentally in [12-15]. The model of spinning gas-film detonation which appeared later [16, 17] contains the following assumptions: combustion begins behind the detonation front of the mixture at a significant distance away from the front, and at the same time the combustion of the entire accumulated fuel mixture occurs instantaneously and all of the stored energy is liberated in the plane of a transverse section of the pipe; the strong explosion wave formed as a result propagates, and decays toward the front. It is proposed in [16, 17] that in a circular pipe, because of the symmetry, the explosion wave will not move along a spiral; during the motion of the detonation front such explosions occur periodically as a fresh mixture is accumulated and periodically accelerate the motion of the forward front of the detonation wave.

In reality, in the experiment the combustion of the mixture occurs gradually and continuously in the entire reaction zone and begins practically immediately behind the detonation front (in a transverse wave acceleration of the combustion occurs only locally); in the rotating system tied to the "head" of the spin, the amplitude of the disturbances is stationary as a function of time and increases toward the front; the detonation wave propagates with a constant velocity; and, under the conditions of spinning detonation, the period-

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dic explosion waves do not react deep in the reaction zone. Aside from these basic contradictions, the model of [16, 17] does not explain the reason for the rotation of the transverse wave and the tail, and it does not permit calculating their form. Based on what was said above, it is necessary to construct a model of spinning gas-film detonation.

2. The change in the spin frequency in the gas-film system, noted above, when concentric inserts are introduced into the pipe [12], indicates that, as in gases, the spin is affected by the acoustic characteristics of the pipe. The experiments performed in [12-15] imply that the transverse wave, gradually transforming into a tail, rotates as a whole along a spiral trajectory with practically uniform pitch along the wall of the pipe and at any fixed moment in time it forms a spiral whose pitch increases away from the front. It is natural to propose that the basic reasons for the formation of the spiral form of a transverse wave and a tail is that the parameters of the flux change along the flow and that the spin frequency ω_s and ω_{11} are not equal.

In this work the frequencies of spinning gas-film detonation are calculated and the effect of the nonuniformity of the parameters of the main flow on the form of the transverse wave and the tail in the subsonic and supersonic regions of the flow behind the detonation front is taken into account.

To describe the gas flow in a system of coordinates fixed on the detonation front, we write out the differential equations for the conservation of the mass, momentum, and energy fluxes taking into account the flow of mass from the film into a distinguished elementary volume, friction, and thermal processes (liberation of heat and heat losses):

$$\frac{\partial}{\partial t}\rho^* + \nabla(\rho^*\mathbf{u}^*) = \dot{M}; \tag{2.1}$$

$$\rho^*\frac{\partial \mathbf{u}^*}{\partial t} + \rho^*(\mathbf{u}^*\cdot\nabla)\mathbf{u}^* = -\nabla p^* + \dot{M}(\mathbf{u}_\omega - \mathbf{u}^*) + \mathbf{F}_\omega; \tag{2.2}$$

$$\frac{\partial}{\partial t}\left[\rho^*\left(e^* + \frac{\mathbf{u}^{*2}}{2}\right)\right] + \nabla\left[\rho^*\mathbf{u}^*\left(e^* + \frac{\mathbf{u}^{*2}}{2}\right)\right] = -\nabla(\mathbf{u}^*p^*) + \dot{M}\left(h_f + H_f + \frac{u_\omega^2}{2}\right) - Q_\omega + \mathbf{F}_\omega\cdot\mathbf{u}_\omega; \tag{2.3}$$

and, the equation of state of the gas

$$p^* = p^*(\rho^*, s^*). \tag{2.4}$$

Here, $\rho^* = \rho^*(\mathbf{x}, t)$, $p^* = p^*(\mathbf{x}, t)$, $\mathbf{u}^* = \mathbf{u}^*(\mathbf{x}, t)$, $s^* = s^*(\mathbf{x}, t)$, $e^* = e^*(\rho^*, s^*)$ are, respectively, the density, pressure, velocity, specific entropy, and internal energy of the particles of the gas occupying the position $\mathbf{x}$ at the time t; $\mathbf{F}_\omega$ is the volume density of the friction force; $\dot{M}$ is the inflow of fuel mass into a unit volume per unit time; H_f and h_f are the liberation of heat per unit weight of the fuel and the specific enthalpy of the fuel; Q_ω is the volume heat flow toward the boundary; the index ω refers to parameters evaluated at the boundary (wall); $u_\omega = U_0$ (U_0 is the velocity of the detonation). The system (2.1)-(2.4) describes a flow in a two-phase gas-film system under the assumption that the parameters $\dot{M}$, $\mathbf{F}_\omega$, Q_ω are averaged over the entire cross section of the pipe.

If, as in [18-20], the flow in the pipe behind the front of the stationary detonation is assumed to be one-dimensional and the parameters of the gas (u_0, ρ_0, e_0, p_0, T_0 – the temperature) are averaged over the cross section of the pipe, then the differential conservation equations can be written in the form

$$\frac{\partial}{\partial z}(\rho_0 u_0) = \dot{M}; \tag{2.5}$$

$$\rho_0 u_0\frac{\partial u_0}{\partial z} = -\frac{\partial}{\partial z}p_0 + F_\omega + \dot{M}(u_\omega - u_0); \tag{2.6}$$

$$\frac{\partial}{\partial z}\left[\rho_0 u_0\left(e_0 + \frac{u_0^2}{2}\right)\right] = -\frac{\partial}{\partial z}(u_0 p_0) + \dot{M}\left(h_f + H_f + \frac{u_\omega^2}{2}\right) - Q_\omega + F_\omega u_\omega; \tag{2.7}$$

and, the equation of state

$$p_0 = \rho_0 R T_0, \tag{2.8}$$

where z is the coordinate oriented away from the front along the flow; $\dot{M} = (k\Pi/A)(\rho u)_\omega$; $F_\omega = (\Pi/A)[(1 - k)\tau_\omega{}^0 + k\tau_\omega]$; $Q_\omega = (\Pi/A)[(1 - k)q_\omega{}^0 + kq_\omega]$; $(\rho u)_\omega$ is the mass flux from a unit surface area per unit time; k is the fraction of the surface covered with the liquid fuel;

Π and A are the perimeter and area of the cross section of the pipe; τ_ω and q_ω are the friction stress and the heat losses per unit surface area (the superscript 0 refers to parameters on the dry wall); and R is the universal gas constant.

In some experiments, states of spinning gas-film detonation with a transverse wave of the acoustic type, which is weak everywhere behind the flow, are observed. If there is a strong transverse wave near the front (the wave of a shock or shock-acoustic type), then its amplitude decays to zero toward the axis of the pipe and eight to ten pipe diameters from the front the wave degenerates everywhere in the volume into a weak acoustic wave [12-15].

Assuming that the pressure disturbances accompanying spinning detonation are small, we shall study the problem of the propagation of small disturbances in an inhomogeneous medium. Let some exact solution of the equations (2.1)-(2.4) – the basic motion – be known: $\rho^* = \rho_0(\mathbf{x}, t)$, $p^* = p_0(\mathbf{x}, t)$, $s^* = s_0(\mathbf{x}, t)$, $u^* = u_0(\mathbf{x}, t)$. We shall seek a solution in the form $\rho^* = \rho_0 + \delta\rho$, $p^* = p_0 + \delta p$, $s^* = s_0 + \delta s$, $u^* = u_0 + \delta u$ [ρ, p, u, and s are small corrections to the main solution (its disturbances), and δ is a parameter].

In what follows we shall always be concerned with the case when the scale of the changes in the parameters of the main flow L is much greater than the scale of the changes in the parameters of the perturbed flow ℓ_*:

$$L \gg l_*. \tag{2.9}$$

The first law of thermodynamics and (2.4) for isentropic disturbed processes ($\delta s \to 0$) or processes such that $c_0^2\delta\rho \gg \frac{\partial e_0}{\partial s_0}\delta s$, $c_0^2\delta\rho \gg \frac{\partial p_0}{\partial s_0}\delta s$ (the velocity of sound $c_0(\mathbf{x}, t) = \sqrt{\left.\frac{dp_0}{d\rho_0}\right|_{s_0=\mathrm{const}}}$) imply that $\delta e \simeq \frac{p_0}{\rho_0^2}\delta\rho$, $\delta p \simeq c_0^2\delta\rho$. Neglecting the effect of perturbed processes on the mass flow rate from the film, friction, and thermal processes (i.e., $\delta\dot{M}$, $\delta\mathbf{F}_\omega$, $\delta Q_\omega \to 0$) and taking into account (2.9), after the standard linearization procedure (in the limit $\delta \to 0$) the system (2.1)-(2.4) assumes the form

$$D_0\rho + \rho_0\nabla\mathbf{u} = 0; \tag{2.10}$$

$$D_0\mathbf{u} + \frac{c_0^2}{\rho_0}\nabla\rho = 0; \tag{2.11}$$

$$p = c_0^2\rho, \tag{2.12}$$

where $D_0 = \partial/\partial t + \mathbf{u}_0\cdot\nabla$.

In deriving (2.11) we employed the following estimate:

$$\left|\frac{\partial\mathbf{u}}{\partial t}\right| \sim \frac{u}{\tau_*}, \quad |\mathbf{u}_0\cdot\nabla\mathbf{u}| \sim \frac{u_0 u}{l_*}, \quad \left|\frac{c_0^2}{\rho_0}\nabla\rho\right| \sim \frac{c_0^2\rho}{\rho_0 l_*}$$

$$\left|\frac{\dot{M}\mathbf{u}}{\rho_0}\right| \sim \frac{\sigma_0^0 U_0}{L}\frac{u}{\rho_0} \sim \frac{uU_0}{Ln}\alpha.$$

Here τ_* is the time scale over which the disturbed parameters change; $\rho_0{}^0$ is the density of the gas in front of the front; $\sigma_0{}^0$, volume density of the liquid phase in the system; $\alpha = \sigma_0{}^0/\rho_0{}^0$, coefficient of stoichiometry ($\alpha < 1$); $n = \rho_0/\rho_0{}^0 > 1$. Since $\ell_* \ll L$, $u_0 \sim 0.2U_0$, the term $\dot{M}u/\rho_0$ in (2.11) [or the term $\dot{M}\delta u$ in the linearized equation (2.2)] can be neglected compared with the remaining terms. Under these assumptions (2.2) and (2.3) are equivalent for weak disturbances. Unlike [1-4], (2.10)-(2.12) take into account the dependence of the parameters of the medium on the coordinates – here there does not exist a coordinate system moving with the gas, in which $u_0(\mathbf{x}, t) = 0$ everywhere.

Applying to (2.11) the curl operation, we obtain, taking into account (2.9), the equation

$$\frac{\partial}{\partial t}\,\mathrm{curl}\,\mathbf{u} = -(\mathbf{u}_0\cdot\nabla)\mathrm{curl}\,\mathbf{u} \tag{2.13}$$

which indicates that in a fixed particle of the medium, moving along the trajectory, $\mathrm{curl}\,\mathbf{u} = \mathrm{const}$. For the case of harmonic oscillations of the particle (which are studied in what follows) the time-averaged value of the velocity u equals zero, so that $\mathrm{curl}\,\mathbf{u} = 0$ and there exists a potential φ such that

$$\mathbf{u} = \nabla\varphi. \tag{2.14}$$

Substituting (2.14), Eqs. (2.10)-(2.12) assumes the form

$$D_0\rho + \rho_0\Delta\varphi = 0,\; D_0\varphi + c_0^2\rho/\rho_0 = 0,$$

whence follows an equation for φ:

$$\frac{\partial^2\varphi}{\partial t^2} = c_0^2\Delta\varphi - 2\mathbf{u}_0\cdot\nabla\frac{\partial\varphi}{\partial t} - \mathbf{u}_0\cdot\nabla(\mathbf{u}_0\cdot\nabla\varphi). \tag{2.15}$$

An equation of this form in a stationary coordinate system for a uniform flow, moving relative to it with a constant velocity $u_0(x, t)$, is presented in [21].

Applying the operations $\partial/\partial t$ and ∇ to (2.10) and the operation ∇ to (2.11), we obtain an equation for ρ analogous to (2.15). The equation for p has the same form, taking into account (2.12).

In a new coordinate system $\tilde{x} = x - V\tau$, $t = \tau$, moving with the velocity V relative to the front, making the tranaformations $\nabla = \tilde{\nabla}$, $\Delta = \tilde{\Delta}$ (the tilde denotes the new variables), $\partial/\partial t = \partial/\partial\tau - V\cdot\tilde{\nabla}$, $\partial^2/\partial t^2 = \partial^2/\partial\tau^2 - \partial V/\partial\tau\cdot\tilde{\nabla} - 2(V\cdot\tilde{\nabla})\partial/\partial\tau + (V\cdot\tilde{\nabla})(V\cdot\tilde{\nabla})$, Eq. (2.15) assumes the form $\frac{\partial^2\varphi}{\partial\tau^2} = c_0^2\tilde{\Delta}\varphi - (\mathbf{u}_0\cdot\tilde{\nabla})(\mathbf{u}_0\cdot\tilde{\nabla})\varphi - 2(\mathbf{u}_0 - \mathbf{V})\cdot\tilde{\nabla}\frac{\partial\varphi}{\partial\tau} + 2(\mathbf{u}_0\cdot\tilde{\nabla})(\mathbf{V}\cdot\tilde{\nabla})\varphi - (\mathbf{V}\cdot\tilde{\nabla})(\mathbf{V}\cdot\tilde{\nabla})\varphi.$ This transformation of coordinates is desirable, for example, in order to reduce the equation of the mixed type obtained in some region to an equation of the elliptic type with $\varphi(x, t) \sim \exp\cdot(i\omega t)$, $i = \sqrt{-1}$. Assuming the main flow behind the front of a stationary detonation wave is one-dimensional, in an inertial coordinate system moving along the axis of the pipe, for steady-state wave phenomena we obtain the equation

$$-\omega^2\varphi = c_0^2\tilde{\Delta}\varphi - (u_0 - V)^2\frac{\partial^2\varphi}{\partial\tilde{z}^2} - 2i\omega(u_0 - V)\frac{\partial\varphi}{\partial\tilde{z}}, \tag{2.16}$$

which is an elliptic equation in the laboratory coordinate system in the region behind the Chapman–Jouguet point, whose solution we seek among the class of smooth functions which are periodic in θ (the angle along a circle). The condition of impenetrability is imposed on the pipe wall ($r = R_0$):

$$\left.\frac{\partial\varphi}{\partial r}\right|_{r=R_0} = 0. \tag{2.16a}$$

On the forward shock front with the form $z = F(r, \theta, t)$, where $F(r, \theta, t) \ll R_0$, from the laws of conservation of mass, momentum, and energy for a curvilinear shock and the conditions of conservation of the tangential components of the velocity we can write the relations

$$\left.\frac{\partial\varphi}{\partial r}\right|_{F+} = g\frac{\partial F}{\partial r},\; \left.\frac{\partial\varphi}{\partial\theta}\right|_{F+} = g\frac{\partial F}{\partial\theta},\; \left.\frac{\partial\varphi}{\partial z}\right|_{F+} = f\frac{\partial F}{\partial t} \tag{2.16b}$$

(the index + indicates a state to the right of the shock). For an ideal gas

$$g = U_0 - u_1,\; f = \frac{M_1^2(\gamma_1 - 1)(U_0 - u_1)}{U_0(1 - M_1^2)}\left[\frac{U_0}{u_1} - \frac{1 + M_1^2\gamma_1}{M_1^2(\gamma_1 - 1)}\right]. \tag{2.16c}$$

Here $M_1^2 = u_1^2/c_1^2$; the index 1 refers to the state behind the planar shock front.

Equation (2.16b) implies that

$$g\left.\frac{\partial^2\varphi}{\partial r\partial z}\right|_{F+} = f\left.\frac{\partial^2\varphi}{\partial r\partial t}\right|_{F+}. \tag{2.16d}$$

In the most general case the boundary conditions are written analogously to (2.16b) and (2.16c), where

$$g = U_0 - u_1,\; f = \frac{\frac{g}{u_1}\left[1 - \frac{u_1}{RU_0}\left(1 + \frac{RT_1}{u_1^2}\right)\frac{\partial I}{\partial T_1}\right]}{1 - \frac{\partial I}{\partial T_1}\frac{1}{R}\left(1 - \frac{RT_1}{u_1^2}\right)} \tag{2.16e}$$

(I is the specific enthalpy of the gas).

We seek the solution of (2.16) in the form $\varphi = R(z)Z(z)\Theta(\theta)\exp(i\omega t)$. Separating the independent variables, for $\Theta(\theta)$ we obtain $\Theta''/\Theta = -m^2$, $\Theta = \sum_m \Theta_m \exp(im\theta)$ (m are integers). The equation for R(r) is

$$R''/R + R'/rR - m^2/r^2 = -\lambda^2,$$

and its solution is $R = R_m J_m(\lambda r)$, where J_m is a Bessel function of the first kind of order m. Taking into account (2.16a), we have

$$J'_m(\lambda_{km}R_0) = 0 \tag{2.17}$$

[λ_{km} is the k-th value of λ_m, satisfying (2.17)].

In the particular cases of single-head spinning detonation (one singularity along the radius and one along the circumference of the pipe, k = m = 1), $\lambda_{11}R_0 = 1.84$. Then from (2.16) we obtain the following equation for Z(z) in the system of the front

$$(1 - M^2) Z''/Z - 2i\omega M c_0^{-1} Z'/Z + (\omega^2/c_0^2 - \lambda_{11}^2) = 0 \tag{2.18}$$

($M = u_0/c_0$ is the Mach number of the flow).

The form of the solution (2.18) is most simply analyzed if it is assumed that there exists a region Ω_0 behind the reaction zone with constant parameters $u_0(z) = u_{00}$, $c_0(z) = c_{00}$. We seek the solution in the form $Z = Z_{00}\exp(i\xi_{11}z)$, substituting which into (2.18) we obtain

$$\xi_{11} = \frac{\omega M_{00}/c_{00} - \sqrt{\omega^2/c_{00}^2 - \lambda_{11}^2(1 - M_{00}^2)}}{1 - M_{00}^2} \tag{2.18a}$$

($M_{00} = u_{00}/c_{00}$). The tail in Ω_0 is a spiral with a uniform pitch $h_0 = 2\pi/\xi_{11}$, and a slope angle ε_0 with respect to the generatrix of the pipe: $\tan\varepsilon_0 = \xi_{11}R_0$. It is obvious that h_0 and ε_0 depend on the coordinate system. Since $\varphi|_{\Omega_0} = \varphi_0 R\Theta \exp[i(\xi_{11}x + \omega t)]$, in a new coordinate system moving parallel to the pipe axis ($\tilde{x} = x - V\tau$) the frequency $\tilde{\omega} = \omega + \xi_{11}V$. This effect is called the acoustic Doppler effect. If $\omega = \omega_{11} = \lambda_{11}c_{00}$, then, as one can see from (2.18a), $\xi_{11} = 0$ — the tail is parallel to the generatrix of the pipe and the frequency of the oscillations is independent of the coordinate system. In the reaction zone, where the flow parameters u_0 and c_0 depend on x, ξ_{11} is not constant and the uniformity of the pitch of the spiral is destroyed.

Equation (2.18) is a second-order equation with a regular singular point z_* such that M = 1 for $z = z_*$. The asterisk here and in what follows denotes a value in the Chapman–Jouguet plane. The frequency ω is a parameter, and the boundary condition on the front (2.16d) is a particular case of the condition imposed on the characteristic values for (2.18).

We introduce the coordinate $x = z_* - z$ (the x axis is oriented from $z = z_*$ toward the front), and we denote the function Z(x) by X(x). Then (2.18) and (2.16d) assume the form

$$X''(1 - M^2) + 2i\omega M c_0^{-1} X' + (\omega^2/c_0^2 - \lambda_{11}^2) X = 0; \tag{2.19}$$

$$X'_f = -i\omega f g^{-1} X_f \tag{2.19a}$$

(the index f indicates that the variable must be evaluated on the forward front of the spin detonation). Near the singular point, confining attention to second-order infinitesimals in x in the series expansion of the terms in Eq. (2.19), we obtain

TABLE 1

N	u(z)	$T_0(z)/T_1 = c_0^2(z)/c_1$
1	$a_1 z + u_1$	$1 + z\tau_1/L_1$
2	$U_0[1 - b_2\exp(a_2 z)]$	$\tau_{02} - \tau_2\exp(\lambda_2 z)$
3	$a_3 z + u_1$	$1 + z\tau_1/L_3$

TABLE 2

M_f	U_0, cm/sec	u_1, cm/sec	c_1, cm/sec	γ_1
4.5	148,088	27,007.8	66,640	1.2992
5.2	171,124	28,917	73,986	1.2901
6	197,451	31,174	82,445	1.2810

$$X = \left[1 - i\left(\lambda_{11}^2 - \omega^2/c_*^2\right)\frac{c_* x}{2\omega} - \frac{p_* x^2}{4\left(\left.\frac{dM}{dx}\right|_* - i\omega/c_*\right)}\right]X_*, \tag{2.20}$$

$$p_* = \frac{i\left(\lambda_{11}^2 - \omega^2/c_*^2\right)c_*}{2\omega}\left[2i\omega\left.\frac{d}{dx}\frac{M}{c}\right|_* - \left(\lambda_{11}^2 - \omega^2/c_*^2\right)\right] + \left.\frac{2\omega^2}{c_*^3}\frac{dc}{dx}\right|_*.$$

3. For calculations of the spatial structure of the disturbance wave it is important to know $u_0(z)$ and $c_0(z)$. Because of the lack of data an uncertainty arises in the description of $\dot{M}$, F_ω, Q_ω in the theoretical determination of the parameters of the main flow, and, in addition, the calculation of $u_0(z)$ and $c_0(z)$ is not the basic problem and makes the calculations too difficult. It is therefore more useful to fix empirical profiles $u_0(z)$ and $c_0(z)$, corresponding satisfactorily to the experimental data [13, 14] (Table 1). Strictly speaking, in this case the functions $u_0(z)$ and $c_0(z)$ must satisfy Eqs. (2.1)-(2.3) or (2.5)-(2.7) with additional conditions imposed on $\dot{M}$, F_ω, and Q_ω. In Table 1, in variants 1 and 2, $\bar{u}(L_1) = u(L_1)/U_0$ and $T_0(L_1)/T_1$, respectively, coincide at $z = L_1$, and in variants 2 and 3 they coincide at $z = z_*$. Taking this into account,

$$a_1 = (u(L_1) - u_1)/L_1, \; b_2 = 1 - \bar{u}_1, \; a_2 = \frac{1}{L_1}\ln\left[\frac{1 - \bar{u}(L_1)}{1 - \bar{u}_1}\right],$$

$$\tau_{02} = 1 + \tau_2, \; \lambda_2 = \frac{1}{L_1}\ln(0{,}1/\tau_2),$$

$$a_3 = (u_* - u_1)/L_1, \; L_3 = \tau_1 z_*/[(c_*/c_1)^2 - 1].$$

The parameters $\bar{u}(L_1)$, $T_0(L_1)/T_1$, $\bar{L}_1 = L_1/R_0$ are given and were varied as follows: $\bar{u}(L_1) =$ 0.7 and 0.8; $\tau_1 = 1$, 1.25, and 1.5; $\tau_2 = \tau_1 + 0.1$; $\bar{L}_1 = 10$-32. Here it is assumed that the liberation of energy continues behind the Chapman–Jouguet plane, and in variants 1 and 3 the quantity z is bounded by the value when $u(z) \le U_0$.

The values of u_1, c_1 and γ_1 in Table 2 are determined from the calculation of the state behind the flat shock front for oxygen, whose properties were taken from [22], for different Mach numbers of the detonation wave ($M_f = U_0/c_0{}^0$, $c_0{}^0 = 329.08$ m/sec is the velocity of sound in front of the wave front). Figure 1 shows the dimensionless variables $\bar{u}_0(z)$ and $\bar{c}_0(z)$ (curves 1-3), corresponding to N = 1-3 (see Table 1) with $M_f = 5.2$, $\bar{u}(L_1) = 0.7$, $\tau_1 = 1$, $\bar{L}_1 = 32$.

Equation (2.19) was solved by the method of finite differences on an M40-30 computer, the first and second derivatives were evaluated using the difference relations

$$X'(x) = \frac{X(x+h) - X(x-h)}{2h}, \quad X''(x) = \frac{X(x+h) - 2X(x) + X(x-h)}{h^2}$$

(h is the step along x), which provide an approximation of second-order accuracy for (2.19); here the mismatch

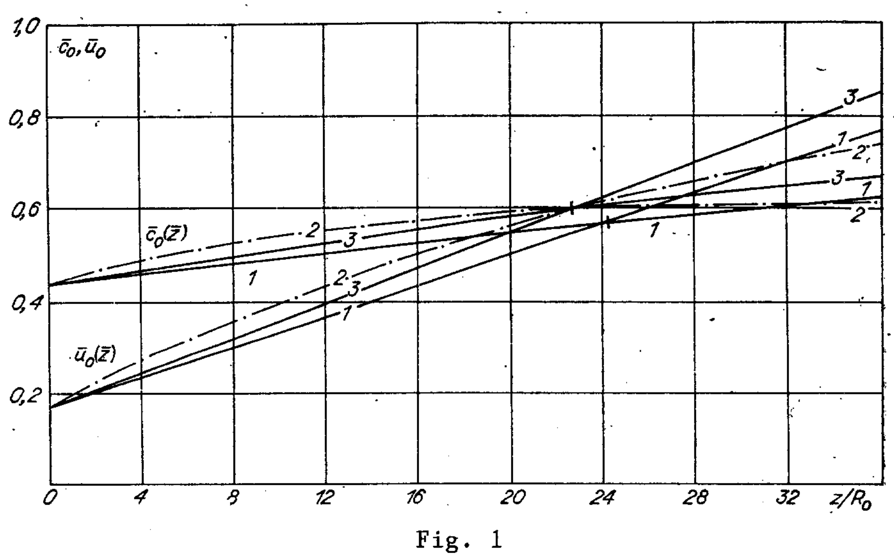

Fig. 1

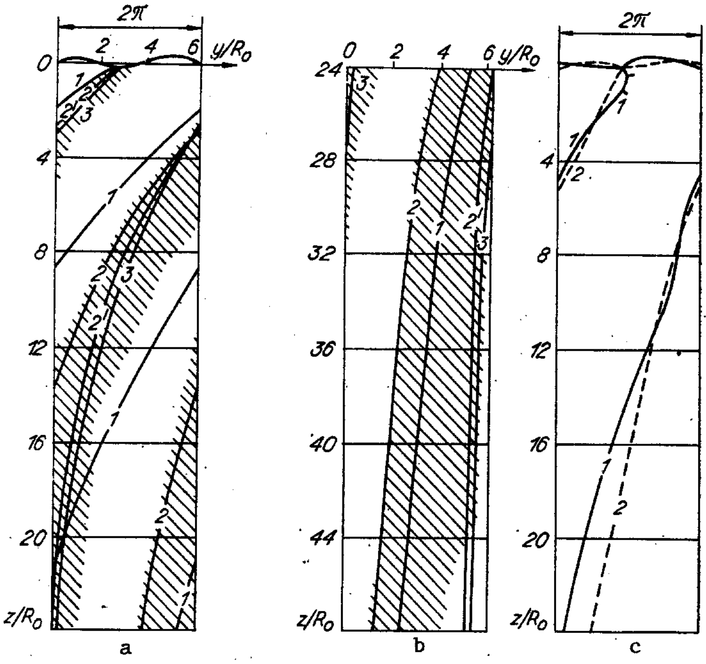

Fig. 2

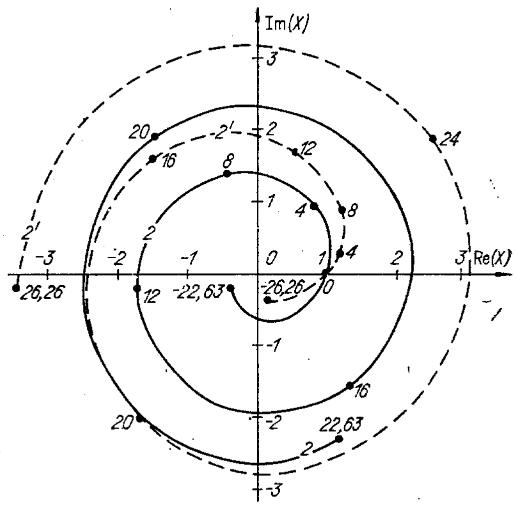

Fig. 3

$$\left| h^2 \frac{(1-M^2)}{12}\frac{d^4X}{dx^4} + \frac{i\omega M}{3c_0} h^2 \frac{d^3X}{dx^3}\right| \leqslant \frac{h^2}{3}\left|\frac{d^3X}{dx^3}\right|$$

The stability of the difference scheme was checked in the calculation by varying h.

For computational convenience at the Chapman–Jouguet point it was assumed that $X_* = 1$, $X(\pm h)$ was calculated using the relation (2.20) and the values of X at subsequent points were determined from the difference equation.

The initial approximation $\omega_0 = \lambda_{11}c_0(z_*)$ was used for the frequency. Each n-th cycle for selecting the next values of ω_n consisted of four calculations through the zone up to the detonation front with values $\omega_n^j = \omega_{n-1} \pm \frac{\omega_0}{2^{n+n_0}}(1 \pm i)$ corresponding to the centers of four squares in one square centered at the point ω_{n-1} ($n_0 \geq 0$ is an integer, $j = 1, \ldots, 4$). Among the four values of ω_n^j for the next approximation a frequency $X_f' = -ifg^{-1}\omega_n X_f$ and X_f', obtained from the numerical calculation. The computational cycles were repeated until an accuracy $\leq 0.5\%$ was achieved.

TABLE 3

M_f	N	τ_1	$\bar{L}_1$	$\bar{u}(L_1)$	$\bar{z}_*$	c_*, m/sec	g/f, m/sec	Re (ωR_0), m/sec	Im (ωR_0), m/sec
4,5	2	1	20	0,7	15,551	925,678	—1309,248	2017,616	—68,196
	2	1	32	0,7	24,88	925,678	—1309,248	2018,032	—37,009
	3	1	32		24,88	925,678	—1309,248	2016,628	—17,933
	2	1	32	0,8	16,196	887,391	—1309,248	2021,267	—38,069
	2	0,5	32	0,8	10,272	756,183	—3879,836 *	1329,407	—15,588
5,2	1	1	10	0,7	15,866	1190,125	—1575	2041,204	—54,532
	1	1	16	0,7	12,21	982,617	—1575	2041,963	—43,258
	1	1	20	0,7	11,178	923,935	—1575	2042,87	—40,675
	2	1	16	0,7	11,32	1019,35	—1550,91	2071,598	—89,75
	2	1	32	0,7	22,626	1019,326	—1550,91	2066,963	—44,874
	2	1,25	32	0,7	26,266	1095,04	—1550,91	2067,995	—53,126
	2	1	32	0,7	22,626	1019,326	—3999,78 *	1500,267	—31,574
	2	0,5	32	0,7	16,192	862,09	—3999,78 *	1499,193	—16,955
	2	0,5	32	0,85	8,764	821,44	—3999,78 *	1502,53	—21,51
6	2	1	20	0,7	13,07	1127,58	—1822,156	2190,233	—79,02
	2	1	32	0,7	20,92	1127,58	—1822,156	2186,69	—49,13
	2	0,5	32	0,8	10,27	926,33	—3879,84 *	1698,51	—24,34

*The boundary condition was calculated for oxygen from (2.16d) taking the properties from the tables of [22], and in the other cases it was calculated from (2.16c); the value of N corresponds to Table 1.

4. The calculations were compared with experiment for the spatial structure (form) of the acoustic wave and the value of the spin frequency. In accordance with the solution of Bessel's equation, the intensity of the wave decays along the radius toward the center of the pipe; along the pipe the crest of the wave is a spiral described by the relation $y = \xi_1(x)R_0$ [the y axis is oriented perpendicular to the x axis, $X = X_0 \exp(i\xi)$, $\xi = \xi_1(x) + i\xi_2(x)$]. The quantity $\exp(-\xi_2(x))$ characterizes the change in the relative amplitude φ of the disturbance wave along the x axis.

Figure 2 shows in the y/R_0, z/R_0 plane (unfolding of the pipe on a surface) examples of calculations (a, b) of the form of the disturbance wave and two structures of spinning gas-film detonation (c) according to the experiments of [13-15] with a transverse wave of the shock 1 and acoustic 2 types. Figure 2b shows a continuation in the coordinate z/R_0 of the curves in Fig. 2a; here, $\bar{L}_1 = 32$, $\bar{u}(L_1) = 0.7$, $\tau_1 = 1$ (except curve 2', where $\tau_1 = 1.25$). Curves 1, 2, and 2' were calculated for N = 2 (see Table 1), for M_f = 4.5, 5.2, and 6, respectively. In Fig. 2a, b, according to [12-15], the shaded region contains diverse experimental structures of spinning detonation, obtained in pipe with diameters ranging from 27 to 70 mm. For convenience the experimental data and all computed curves in Fig. 2 were constructed so as to emanate from one point on the central line of the forward front z = 0. In these and other variants of the calculations the theory agrees well not only qualitatively but also quantitatively with experiments.

In Fig. 3 the curves $X(x) = \mathrm{Re}(X(x)) + i\,\mathrm{Im}(X(x))$ with $-z_* \le x \le z_*$, characterizing simultaneously the form of the crest of the disturbance wave, and its amplitude, were constructed for 2 and 2' (see Fig. 2a, b). The numbers on the points indicate the corresponding values of x/R_0; in the Chapman–Jouguet plane $X(0) = 1$. It is evident that, as in the experiments, the amplitude of the disturbance increases toward the forward front of the detonation wave.

The values of ω calculated for a number of variants are presented in Table 3, whence it is evident that the spin frequency $\omega_S = \mathrm{Re}(\omega)$ is virtually independent of the profiles of the flow parameters in the reaction zone, and the boundary conditions on the front g/f have an appreciable effect on the value of ω_S. $\mathrm{Re}(\omega)$ is virtually independent of the accuracy of the expansion and of the calculation in the vicinity of the singular point; for example, if $X(\pm h)$ is evaluated according to $X(\pm h) = X_*$ rather than (2.20). The profiles of the main flow affect the value of $\mathrm{Im}(\omega)$, which determines the growth in the amplitude of the disturbance as a function of time.

The frequency ω_s can be evaluated from the expression

$$\tilde{\omega}_s = \frac{\lambda_{11}c_1}{\sqrt{(1+u_1 f/g)^2 - f^2 c_1^2/g^2}}, \tag{4.1}$$

which agrees to within less than 1% with Re (ω) from Table 3 and is obtained from (2.18) under the assumption that near the detonation front $\xi = k_1 z$ (the linear approximation holds). Then on the front for (2.18) we can write the boundary conditions

$$Z'_f = ifg^{-1}\tilde{\omega}_s Z_f, \; Z''_f = -f^2\tilde{\omega}_s^2 g^{-2} Z_f,$$

which are conditions on $\tilde{\omega}_s$. To accelerate the process of finding a solution ω, in a number of variants the initial approximation $\omega_0 = \tilde{\omega}_s$ was given.

The spin frequency ω_s = Re (ω) agrees better with the experimental values of the frequency $\omega_{es} = (1\text{-}1.6)\cdot 10^3$ m/sec$\cdot R_0^{-1}$ (m) [12-15] in the case when the boundary conditions are calculated using the relation (2.16d) (see Table 3). The frequencies $\omega_{11} = \lambda_{11}c_*$, calculated based on the acoustic theory proposed in [1-4], differ insignificantly (on the average by 10-15%) from Re (ω). The surprisingly close agreement between ω_{11} calculated exactly and experiments indicates not only the correctness of the method developed in [1-4], but also the fact that ω_s depends on the resonance characteristics of the transverse section of the pipe, determining λ_{11}, and the velocity of sound, which varies insignificantly behind the front. This can be seen from Fig. 1 and a comparison of the dependences (4.1) and $\omega_{11} = \lambda_{11}c_*$.

The approach developed in this work enables establishing the basic properties of spinning detonation, calculating the spin frequency, and the form of the disturbance wave in the entire region of the flow behind the detonation front. It was shown that the spin frequency depends primarily on the boundary conditions on the forward front. When the assumptions made above hold, the results obtained can be transferred to other systems (for example, gas-droplet systems).

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ISOTHERMS AND GRÜNEISEN FUNCTIONS FOR 25 METALS

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1. Introduction. The study of the compressibility of materials at 0°K and at room temperatures is traditionally one of the main directions of research in high-pressure physics. In works devoted to this problem the sources of experimental information are "normal" isotherms recorded on static setups at T = 293°K, ultrasonic data, and especially shock-wave determinations of the Hugoniot adiabat. Based on dynamic experiments, the zero cold-compression isotherms $p_C(\rho)$ (p_C is the pressure at T = 0; ρ is the density) are found [1, 2] by separating the shock pressures into thermal and "cold" components under acceptable assumptions about the Grüneisen functions, characterizing the thermal elasticity of compressed bodies. In the widely acclaimed work [3], the isotherms of 14 metals are calculated within the framework of the Mie–Grüneisen equations of state according to the parameters of precisely measured Hugoniot adiabats and the approximate relation $\gamma\rho = \gamma_0\rho_0$ (γ is the temperature-independent Grüneisen constant). In this manner, in particular, the standard isotherms of copper molybdenum, silver and palladium, used in [4] for calibrating fluorescence ruby pressure gauges for the megabar range, were calculated. In [5-8], more complete equations of state, including the electronic components and taking into account the anharmonicity of the vibrations of the crystal lattice, were employed for the interpretation of dynamic experiments for the same purposes. The "harmonic" Grüneisen coefficients were calculated here based on different variants of the theory of small vibrations which, however, do not have any strict justifications.

Another method for constructing the curves $p_C(\rho)$ of the potential interaction and the normal isotherms $p_T(\rho)$ is based on the determination of the parameters of semiempirical potentials from the isentropic κ_0S or isothermal κ_0T bulk moduli of the initial state (T = 293°K, p = 0) and their derivatives with respect to the pressure — κ_1S or κ_1T. The values of κ_0S and κ_0T were found in [9-11] by means of analytic approximations of the isotherms, recorded up to p = 4.5 GPa. Different methods of approximation yielded stable values of κ_0T and very different derivatives κ_1T, which determined the extrapolation behavior of the curves. A somewhat better results using the same information was achieved in [12] in the description of isotherms by the Morse potential and by taking into account the sublimation energy, which had a stabilizing effect.

The isentropic characteristics of the initial compressibility and their isothermal analogs are revealed with high reliability by ultrasonic measurements at atmospheric and high

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